The Development of an Energy-Based Nearfield Acoustical Holography Technique

Scott R. Woolston
Department of Mechanical Engineering
Brigham Young University
435 CTB
Provo, UT 84604

Abstract

Nearfield Acoustical Holography (NAH) is an acoustic imaging technique that has been traditionally based on pressure measurements. A NAH technique has been developed that allows for energy density (ED) to be propagated. However, because ED does not satisfy the wave equation, it could not be propagated directly. ED is propagated through the individual propagations of pressure and particle velocity measurements and then combined to form ED on the reconstruction plane.

Introduction

In 1980, Williams, Maynard, and Skudrzyk[1] developed an acoustical imaging technique known as nearfield acoustical holography (NAH).[2,3] It has proven to be a useful tool for measuring velocity and energy distributions on surfaces, as well as characterizing noise sources.[4,5] Conventional NAH relies on pressure measurements made in the acoustic nearfield where propagating and evanescent waves are both present. Propagating waves radiate well and transmit energy to the farfield. Evanescent waves radiate poorly and decay exponentially, never leaving the acoustic nearfield.[6] Because the measurements are performed in the nearfield, before the evanescent waves have completely decayed, sub-wavelength resolution on the reconstruction surface can be obtained.

In conventional NAH, this resolution comes, however, at the cost of a dense measurement array. However, recent work by Harris developed an approach that reduces the number of measurement positions required without sacrificing resolution on the reconstruction surface.[7] Harris' approach uses in-plane velocity measurements in addition to traditional pressure measurements. The in-plane velocity measurements are used in conjunction with Euler's equation to relate the particle velocity to the pressure gradient as shown in Eq. (1)

$$\rho_0 \frac{d\vec{u}}{dt} = -\nabla p \tag{1}$$

This gradient was then used with bicubic Hermite surfaces to interpolate the pressure field between measurement points. This resulted in an approximate reduction of 75% in required measurement points for a two-dimensional field without sacrificing accuracy on the reconstruction surface.

Recently, Jacobsen and Liu developed a NAH technique based on the propagation of normal particle velocity only.[8] It was found that NAH based on normal particle velocity was just as accurate as traditional NAH.

The pressure and velocity measurements Harris used are closely related to an energy density (ED) measurement. ED is a measure of the potential and kinetic energy in a sound field. The potential and kinetic energies are proportional to the square pressure and square velocity respectively as shown in Eq. (2), where ρ_0 is the density, u is the particle velocity, p is the acoustic pressure, and c is the acoustic wave speed.[9]

$$ED = \frac{1}{2}\rho_0 \left[u^2 + \left(\frac{p}{\rho_0 c} \right)^2 \right] \tag{2}$$

This leads to the possibility of creating a NAH measurement system where ED is the propagated quantity. Harris, in contrast, used the in-plane velocity components only to obtain a better estimate of the pressure field. The pressure field was then propagated as it is with traditional NAH.

The development of an energy-based NAH (ENAH) technique would require the development of a propagator for ED. A propagator is essentially a transfer function that relates an acoustic quantity at one point in space to another acoustic quantity at another point in space. The pressure propagator used in traditional NAH relates the normal particle velocity at one point to the pressure at another. This propagator was developed by Rayleigh as part of his first integral formula. A similar process will be followed to develop an ED propagator that relates normal particle velocity at one point in space to ED at another. Once this propagator is developed, it will be used to create an ENAH system.

Background

In order to fully comprehend the scope of this research, it is necessary to have a basic understanding of the theory behind NAH. NAH is based on the fact that the acoustic pressure on any plane, z_h , located in the half-space above a source is related to the pressure on another plane, z, by

$$P(k_x, k_y, z) = P(k_x, k_y, z_h) e^{i k_z (z - z_h)} \tag{3}$$

where P is the spatial Fourier transform of the pressure and is known as the angular spectrum, and k_x and k_y are the wave numbers in the x and y directions.[10] The normal particle velocity on the new plane, z, is

$$\dot{W}(k_x, k_y, z) = \frac{k_z}{\rho_0 c k} e^{i k_z (z - z_h)} P(k_x, k_y, z_h) = G(k_x, k_y, z - z_h) P(k_x, k_y, z_h) \tag{4}$$

where

$$G(k_x, k_y, z - z_h) \equiv \frac{k_z}{\rho_0 c k} e^{i k_z (z - z_h)} \tag{5}$$

G is called the velocity propagator because it propagates the velocity on the surface z_h out to pressure on the surface z. When $z \geq z_h$, this represents the forward propagating condition. When $z < z_h$, this represents the inverse problem and G becomes the inverse pressure propagator.

Let the plane $z = z_h$ be the measurement plane and let $z = z_s$ be the surface of the vibrating object. Using the inverse velocity propagator from above, NAH allows a measured pressure distribution to be propagated back to the normal velocity distribution on the vibrating surface as summarized in Eq. (6).

$$\dot{w}(x, y, z_s) = F_x^{-1} F_y^{-1} [F_x F_y [p(x, y, z_h)] G(k_x, k_y, z_s - z_h)] \tag{6}$$

F and F^{-1} represent the Fourier and inverse Fourier transforms respectively.

In words, Eq. (6) can be stated :

- measure the pressure
  $\rightarrow p(x, y, z_h)$
- compute its angular spectrum
  $\rightarrow P(k_x, k_y, z_h)$
- multiply by the inverse propagator
  $G(k_x, k_y, z_s - z_h) \rightarrow \dot{W}(k_x, k_y, z_s)$
- compute the inverse transform
  $\rightarrow \dot{w}(x, y, z_s)$

This is the conventional NAH approach as presented by numerous researchers. The goal of this research is to expand on the conventional NAH approach and develop an ED propagator, G_{ED}, that will be able to provide the relationship

$$\dot{w}(x, y, z_s) = F_x^{-1} F_y^{-1} [F_x F_y [ED(x, y, z_h)] G_{ED}(k_x, k_y, z_s - z_h)] \tag{7}$$

This relationship will be the foundation of the new ENAH system.

Propagator Development

As stated above, NAH is possible because of the relationship given in Eq. (3). This same relationship can be stated for particle velocity

$$V(k_x, k_y, z) = V(k_x, k_y, z_h) e^{i k_z (z - z_h)} \tag{8}$$

where V is the particle velocity in any of the three orthogonal directions. This relationship was of basis of the work done by Jacobsen and Liu. In general such a relationship can be found for any quantity that satisfies the wave equation

$$\nabla^2 p = \frac{1}{c^2} \frac{\partial^2 p}{\partial t^2} \tag{9}$$

in the time domain or the Helmholtz equation

$$\nabla^2 \boldsymbol{\psi} + k^2 \boldsymbol{\psi} = 0 \tag{10}$$

in the frequency domain, where $\boldsymbol{\Psi}$ is a function only of position. The challenge lies in the fact that ED does not satisfy the wave equation. Therefore, there is no propagator that will propagate ED directly. In other words

$$ED(k_x, k_y, z) \neq ED(k_x, k_y, z_h) e^{i k_z (z - z_h)} \tag{11}$$

This means that an alternative method for propagating ED must be developed.

As shown above in Eq. (2), ED is a nonlinear combination of pressure and velocity. It has already been shown that both pressure and velocity satisfy the wave equation, they can both be propagated separately. If pressure and each of the three velocity components can be propagated independently, they can be recombined on the reconstruction surface to recreate ED.

The proposed method requires pressure, and each of the three velocity components to be measured, transformed, propagated, and inverse transformed independently as shown below.

$$p(x,y,z_s)=F_x^{-1}F_y^{-1}[F_xF_y[p(x,y,z_h)]G'(k_x,k_y,z_s-z_h)] \quad (12)$$

$$v_x(x,y,z_s)=F_x^{-1}F_y^{-1}[F_xF_y[v_x(x,y,z_h)]G'(k_x,k_y,z_s-z_h)] \quad (13)$$

$$v_y(x,y,z_s)=F_x^{-1}F_y^{-1}[F_xF_y[v_y(x,y,z_h)]G'(k_x,k_y,z_s-z_h)] \quad (14)$$

$$v_z(x,y,z_s)=F_x^{-1}F_y^{-1}[F_xF_y[v_z(x,y,z_h)]G'(k_x,k_y,z_s-z_h)] \quad (15)$$

where

$$G'(k_x,k_y,z_s-z_h)\equiv e^{ik_z(z-z_h)} \quad (16)$$

The results are substituted into Eq. (2) with

$$u^2=v_x^2+v_y^2+v_z^2 \quad (17)$$

It should be noted that an energy density measurement cannot be propagated directly to normal velocity component, *w*, as is done with traditional NAH. This is due to the fact that there is no direct relationship between normal velocity and ED.

Results

The ENAH method described in the previous section was analytically modeled to verify its validity. A point-driven, simply-supported plate was modeled as shown in figure 1.

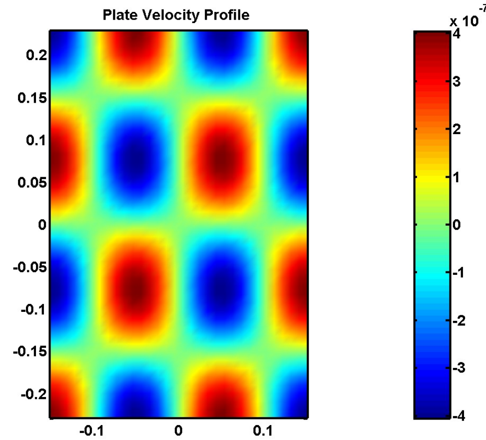

Figure 1 Plate Velocity Profile

Using this velocity information, two closely-spaced pressure fields were propagated a distance of 5 cm away from the plate using Rayleigh's Integral

$$p=\iint_s \hat{u}_n \frac{e^{-ikR}}{R}\, ds \quad (18)$$

Theses two pressure fields were used to calculate the pressure and velocity components needed to calculate ED. The resulting ED field is shown in figure 2.

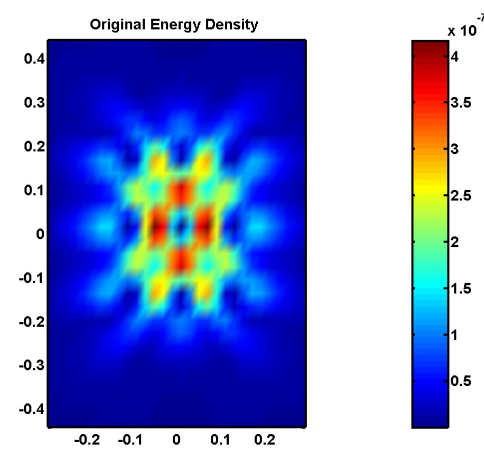

Figure 2 Original Energy Density

The pressure and velocity components were then propagated 2 cm back toward the plate according to Eqns. (12-15) and combined to form an new ED field shown in figure 3.

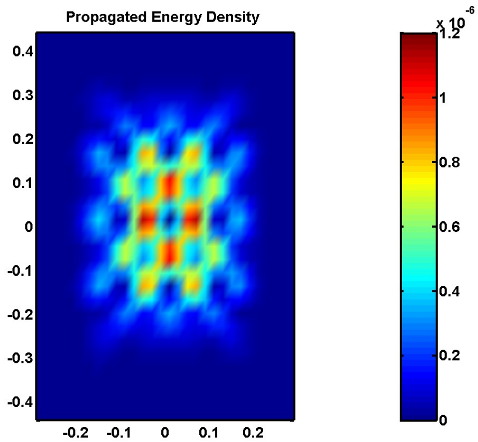

Figure 3 Propagated Energy Density

To measure the performance of ENAH algorithm, a reference ED field was created. An ED field was created by the same method used to create the original field in figure 2. The field was calculated the same distance from the plate as the propagated ED field. The reference field is shown in figure 4.

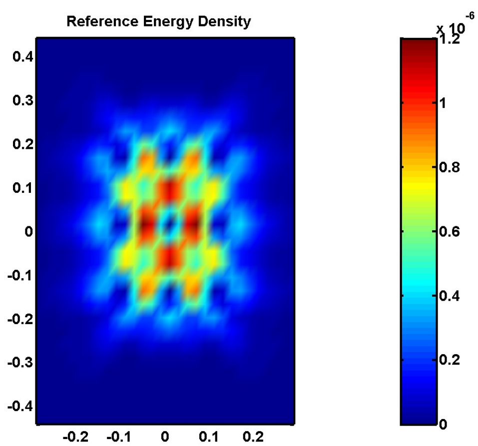

Figure 4 Reference Energy Density Field

The residual between the reference and the propagated ED fields is shown in figure 5.

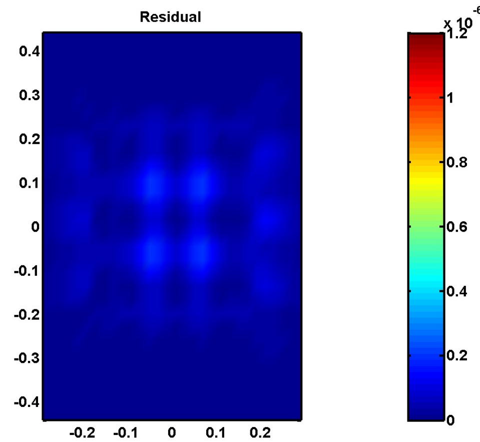

Figure 5 Residual

As shown in figure 5, the residual error resulting from the propagation is very small, near zero at most locations.

Discussion of Results

As shown in the above, the ENAH algorithm performed very well, with an accurate reconstruction of the ED field. It appears that ENAH is a viable method for propagating ED without the existence of a true ED propagator. More work needs to be done to find what can be done to reduce the error in the propagation even further. Experimental validation also needs to be performed.

References:

1 E. G. Williams, J.D. Maynard, and E. Skudrzyk, "Sound source reconstruction using a microphone array," J. Acoust. Soc. Am. 68, 340-344 (1980).

2 J. D. Maynard, E. G. Williams, and Y. Lee, "Nearfield acoustic holography: I. Theory of generalized holography and the development of NAH," J. Acoust. Soc. Am. 78, 1395-1413 (1985).

3 W. A. Veronesi and J. D. Maynard, "Near field acoustic holography (NAH): II. Holographic reconstruction algorithms and computer implementation," J. Acoust. Soc. Am. 81, 1307-1322 (1987).

4 T. Antoine, J. Catalfaud, and C. Aujard, "Meeting Noise Regulations with Nearfield Acoustic Holography," *Sound and Vibration*, 18-20 (2003).

5 D. Vaucher de la Croix, P. Chevret, F. Perrin, "Use of Acoustical Holography in 3D Interiors Measurements," *Proceedings of the International Congress and Exposition on Noise Control Engineering*, August 19-

21, Dearborn, MI. 2002.

6 F. Fahy, *Sound and Structural Vibration: Radiation, Transmission and Response* (Academic Press, San Diego, 1985).

7 M. C. Harris, J. D. Blotter, and S. D. Sommerfeldt, “Obtaining the complex pressure field at the hologram surface for use in near-field acoustical holography when pressure and in-plane velocities are measured,” J. Acoust. Soc. Am. 119, 808-816 (2006).

8 F. Jacobsen, Y. Liu, “Near field acoustic holography with particle velocity transducers,” J. Acoust. Soc. Am. 118, 3139-3144 (2005).

9 P. M. Morse and K. U. Ingard, *Theoretical Acoustics* (McGraw Hill Book Company, 1968).

10 E. G. Williams, *Fourier Acoustics: Sound Radiation and Nearfield Acoustical Holography* (Academic Press, San Diego, 1999).